Inverse of a Matrix and Application of Determinants & Matrix

6 Marks Questions

1. Two schools *P* and *Q* want to award their selected students on the values of discipline, politeness and punctuality. The school *P* wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 1000. School *Q* wants to spend ₹ 1500 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 600, using matrices, find the award money for each value.
Apart from the above three values, suggest one more value for awards.

Value Based Question; Delhi 2014

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}$$

(1)

Now, the solution of given system is given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 2000 + 1500 - 3000 \\ 1000 - 3000 + 3000 \\ -3000 + 1500 + 3000 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 500 \\ 1000 \\ 1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$ **(1)**

On comparing corresponding elements, we get $x = 100$, $y = 200$ and $z = 300$

Honesty is one more value which is also considered for the award. **(1)**

2. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values of 3, 2 and 1 students, respectively with a total award money of ₹ 1600. School B wants to spend ₹ 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award. **All India 2014; Value Based Question**

Let the amount awarded to the students on th values of sincerity, truthfulness an helpfulness be ₹ x, ₹ y and ₹ z, respectively Then, according to the question,

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
and $$x + y + z = 900$$

This system of equations can be written a $AX = B$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

Here, $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$$= 3(1-3) - 2(4-3) + 1(4-1)$$
$$= 3(-2) - 2(1) + 1(3) = -6 - 2 + 3$$
$$= -8 + 3 = -5 \neq 0$$ **(1)**

Thus, A is non-singular matrix.

$\therefore$ A^{-1} exists.

Cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = (1)(-2) = -2$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 3 \\ \end{vmatrix} = (-1)(1) = -1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} & \\ 1 & 1\end{vmatrix} = (-1)(1) = -1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 1 \\ 1 & 1\end{vmatrix} = (1)(3) = 3$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 1 \\ 1 & 1\end{vmatrix} = (-1)(1) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 1 \\ 1 & 1\end{vmatrix} = (1)(2) = 2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 2 \\ 1 & 1\end{vmatrix} = (-1)(1) = -1$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 1 \\ 1 & 3\end{vmatrix} = (1)(5) = 5$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 1 \\ 4 & 3\end{vmatrix} = (-1)(5) = -5$$

and $A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 2 \\ 4 & 1\end{vmatrix} = (1)(-5) = -5$ **(1)**

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Then, $A^{-1} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$$\left[\because A^{-1} = \frac{1}{|A|}\text{adj}(A)\right]$$

$$= \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \qquad \textbf{(1)}$$

Now, given system has a unique solution given by

$$X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3200 + 2300 - 4500 \\ 1600 - 4600 + 4500 \\ -4800 + 2300 + 4500 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 1000 \\ 1500 \\ 2000 \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

On comparing the corresponding elements, we get

$x = 200, y = 300, z = 400$ **(1)**

Hence, the amount of money for each value sincerity, truthfulness are helpfulness are ₹ 200, ₹ 300 and ₹ 400, respectively. **(1)**

Apart from these three values, punctuality should be considered for the award. **(1)**

3. Two schools P and Q want to award their selected students on the values of tolerance, kindness and leadership. The school P wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with a total award money of ₹ 2200. School Q wants to spend ₹ 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is ₹ 1200, using matrices, find the award money for each value.

Apart from these three values, suggest one more value which should be considered for award. **Foreign 2014; Value Based Question**

Let the amount awarded to the students on the values of Tolerance, kindness and leadership be ₹ x, ₹ y and ₹ z respectively, then according to the question,

$$3x + 2y + z = 2200$$
$$4x + y + 3z = 3100$$
$$x + y + z = 1200$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$ **(1)**

Here, $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$

$$= 3(1-3) - 2(4-3) + 1(4-1)$$ **(1)**

$$= -6 - 2 + 3$$
$$= -5 \neq 0$$

So, A is non-singular and its inverse exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 (1-3) = -2$$
$$A_{12} = (-1)^3 (4-3) = -1$$
$$A_{13} = (-1)^4 (4-1) = 3$$
$$A_{21} = (-1)^3 (2-1) = -1$$
$$A_{22} = (-1)^4 (3-1) = 2$$
$$A_{23} = (-1)^5 (3-2) = -1$$
$$A_{31} = (-1)^4 (6-1) = 5$$
$$A_{32} = (-1)^5 (9-4) = -5$$
$$A_{33} = (-1)^6 (3-8) = -5$$ **(1)**

$$\therefore \text{ adj}(A) = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 & -5 \end{bmatrix} \quad \begin{bmatrix} 3 & -1 & -5 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

$$= \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix} \qquad \textbf{(1)}$$

Now, the solution of given system is given by

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 & -5 \\ 1 & -2 & 5 \\ -3 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1500 \\ 2000 \\ 2500 \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix} \qquad \textbf{(1)}$$

On comparing the corresponding elements, we get $x = 300, y = 400$ and $z = 600$

Apart from these three values, sincerity should be considered for the award. **(1)**

4. A total amount of ₹ 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$ respectively. The total annual interest from these three accounts is ₹ 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. **Delhi 2014C**

Let ₹ x, ₹y and ₹z be invested in saving accounts at the rate of 5%, 8% and $8\frac{1}{2}$%, respectively.

Then, the system of equations is

$$x + y + z = 7000 \quad \text{...(i)}$$

and $\frac{5x}{100} + \frac{8y}{100} + \frac{17z}{200} = 550$

$$\Rightarrow \quad 10x + 16y + 17z = 110000 \quad \text{...(ii)}$$

and $$x - y = 0 \quad \text{...(iii)}$$

This system of equation can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $B = \begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$

Now, $|A| = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 16 & 17 \\ 1 & -1 & 0 \end{bmatrix}$ **(1)**

$\Rightarrow \quad |A| = 1(0 + 17) - 1(0 - 17) + 1(-10 - 16)$

$= 17 + 17 - 26 = 8 \neq 0$ **(1)**

$\therefore A^{-1}$ exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 16 & 17 \\ -1 & 0 \end{vmatrix} = 1(0 + 17) = 17$$

$$A_{12} = (-1)^3 \begin{vmatrix} 10 & 17 \\ 1 & 0 \end{vmatrix} = -1(0 - 17) = 17$$

$$A_{13} = (-1)^4 \begin{vmatrix} 10 & 16 \\ 1 & -1 \end{vmatrix} = 1(-10 - 16) = -26$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -1(0 + 1) = -1$$

$$A_{22} = (-1)^4 \begin{vmatrix} & \\ 1 & 0 \end{vmatrix} = 1(0-1) = -1$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1(-1-1) = 2$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 16 & 17 \end{vmatrix} = 1(17-16) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 10 & 17 \end{vmatrix} = -1(17-10) = -7$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 10 & 16 \end{vmatrix} = 1(16-10) = 6 \quad \textbf{(1)}$$

$$\therefore \text{ adj}(A) = \begin{bmatrix} 17 & 17 & -26 \\ -1 & -1 & 2 \\ 1 & -7 & 6 \end{bmatrix}^T = \begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}$$

Then, $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix} \quad \textbf{(1)}$$

The solution of given system is given by

$X = A^{-1} \cdot B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 17 & -1 & 1 \\ 17 & -1 & -7 \\ -26 & 2 & 6 \end{bmatrix}\begin{bmatrix} 7000 \\ 110000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 9000 \\ 9000 \\ 38000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix} \quad \textbf{(1)}$$

On comparing the corresponding elements, we get $x = 1125$, $y = 1125$, $z = 4750$.

Hence, the amount deposited in each type of account is ₹1125, ₹1125 and ₹ 4750, respectively. **(1)**

5. Two schools, P and Q, want to award their selected students for the values of sincerity, truthfulness and hard work at the rate of ₹ x, ₹ y and ₹ z for each respective value per student. School P awards its 2, 3 and 4 students on the above respective values with a total total prize money of ₹ 4600. School Q wants to award its 3, 2 and 3 students on the respective values with a total award money of ₹ 4100. If the total amount of award money for one prize on each value is ₹ 1500, using matrices find the award money for each value. Suggest one other value which the school can consider for awarding the students. **All India 2014C; Value Based Question**

Let the amount awarded to the students on the values of sincerity, truthfulness and hard work be ₹ x, ₹ y and ₹ z respectively, then according to question

$$2x + 3y + 4z = 4600$$
$$3x + 2y + 3z = 4100$$
$$x + y + z = 1500$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4600 \\ 4100 \\ 1500 \end{bmatrix}$$ **(1)**

$$\text{Here, } |A| = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= 2(2-3) - 3(3-3) + 4(3-2)$$
$$= -2 - 0 + 4 = 2 \neq 0$$ **(1)**

So, A is non-singular and its inverse exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 (2-3) = -1$$
$$A_{12} = (-1)^3 (3-3) = 0$$
$$A_{13} = (-1)^4 (3-2) = 1$$
$$A_{21} = (-1)^3 (3-4) = 1$$
$$A_{22} = (-1)^4 (2-4) = -2$$
$$C_{23} = (-1)^5 (2-3) = 1$$
$$A_{31} = (-1)^4 (9-8) = 1$$
$$A_{32} = (-1)^5 (6-12) = 6$$

$$A_{33} = (-1)^6\,(4-9) = -5 \qquad \textbf{(1)}$$

$$\therefore \text{adj}\,(A) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \\ 1 & 6 & -5 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}$$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj}\,A) = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}$ **(1)**

Now, the solution of given system is given by

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{bmatrix}\begin{bmatrix} 4600 \\ 4100 \\ 1500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1000 \\ 800 \\ 1200 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 400 \\ 600 \end{bmatrix}$$

On comparing the corresponding elements, we get $x = 500$, $y = 400$ and $z = 600$. **(1)**

Apart from these three values, punctuality should be considered for the award. **(1)**

6. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47000. If all the three prizes per person together amount to ₹ 12000, then using matrix method, find the values of x, y and z. What values are described in this question?

Delhi 2013C; Value Based Question

Given, two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z, respectively.

Then, according to the given condition,

$$4x + 3y + 2z = 37000$$

$$5x + 3y + 4z = 47000$$

and $$x + y + z = 12000 \qquad \textbf{(1)}$$

The system of equations can be written in matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

i.e. $AX = B$, where ...(i)

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \textbf{(1)}$$

Now, $|A| = 4(3-4) - 3(5-4) + 2(5-3)$

$= -4 - 3 + 4 = -3 \neq 0$

So, A is non-singular and its inverse exists.
Now, cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = +(3-4) = -1$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 4 \\ 1 & 1 \end{vmatrix} = -(5-4) = -1$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = +(5-3) = 2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -(3-2) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 4 & 2 \end{vmatrix} = +(4-2) = 2$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 4 & 3 \end{vmatrix} = -(4-3) = -1$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 4 & 2 \\ 5 & 4 \end{vmatrix} = -(16-10) = -6$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 4 & 3 \\ 5 & 3 \end{vmatrix} = +(12-15) = -3 \quad \textbf{(1)}$$

$$\therefore \text{adj }(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

Then, $$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix} \quad \textbf{(1)}$$

Now, solution of Eq. (i) is given by $X = A^{-1}B$

$$\therefore \quad X = -\frac{1}{3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -37000 - 47000 + 72000 \\ -37000 + 94000 - 72000 \\ 74000 - 47000 - 36000 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 4000, y = 5000 \text{ and } z = 3000 \quad \textbf{(1)}$$

The value described in the question are resourcefulness, competence and determination. **(1)**

7. A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of ₹ 6000. Three times the award money for hard work added to that given for honesty, amounts to ₹ 11000. The award money given for honesty and hard work together is double the one given for regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, honesty, regularity and hard work, suggest one more value which the school must include for award.

Value Based Question; Delhi 2013

> Consider x, y and z are the award of honesty, regularity and hard work and form the system of equations. Then, write them in matrix form as $AX = B$. Now, the solution is given by $X = A^{-1}B$, put the values of A^{-1}, X and B and calculate the required values.

Let award for honesty = ₹ x

Award for regularity = ₹ y

and award for hard work = ₹ z

According to first condition,

$$x + y + z = 6000$$

According to second condition,

$$3z + x = 11000$$

According to third condition,

$$x + z = 2y$$

Now, the above equations can be rewritten in standard form of linear equations as

$$x+y+z=6000, \quad \text{...(i)}$$

$$x+0y+3z=11000 \quad \text{...(ii)}$$

and $$x-2y+z=0 \quad \text{...(iii)} \ \mathbf{(1)}$$

We can represent these equations using matrices as

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}$$

i.e. $AX=B$,

where, $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}6000\\ 11000\\ 0\end{bmatrix}$

and its solution is given by

$$X=A^{-1}B \quad \text{...(iv)}$$

Now, $|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{vmatrix}$

$$=1(0+6)-1(1-3)+1(-2-0)$$

$$=6+2-2=6\neq 0 \quad \mathbf{(1)}$$

$\therefore A^{-1}$ exists, because A is a non-singular matrix.

Now, cofactors of $|A|$ are

$$A_{11}=(-1)^{1+1}\begin{vmatrix}0 & 3\\ -2 & 1\end{vmatrix}=+(0+6)=+6$$

$$A_{12}=(-1)^{1+2}\begin{vmatrix}1 & 3\\ 1 & 1\end{vmatrix}=-(1-3)=2$$

$$A_{13}=(-1)^{1+3}\begin{vmatrix}1 & 0\\ 1 & -2\end{vmatrix}=+(-2)=-2$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} \; & \; \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = +(1-1) = 0$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = +(3-0) = 3$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = +(0-1) = -1$$

Now, $C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}$

(1)

$\therefore \operatorname{adj}(A) = C^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$

Then, $A^{-1} = \frac{1}{|A|}(\text{adj}) = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$ **(1)**

Now, $X = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$

$$= \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 - 0 \\ -12000 + 33000 - 0 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

On comparing, we get

$$x = 500, y = 2000 \text{ and } z = 3500$$

Hence, award for honesty = ₹ 500

Award for regularity = ₹ 2000

and award for hard work = ₹ 3500 **(1½)**

The school must include punctuality for award. **(1/2)**

8. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Value Based Question; All India 2013

Given,

Number of members for honesty = x

Number of members for helping others = y

and number of members for supervising the workers to keep the colony neat and clean = z

Now, by first condition,

$$x + y + z = 12 \quad \text{...(i)}$$

By second condition,

$$2x + 3(y + z) = 33$$

$$\Rightarrow \quad 2x + 3y + 3z = 33 \quad \text{...(ii)}$$

and by third condition,

$$(x + z) = 2y$$

$$\Rightarrow \quad x - 2y + z = 0 \quad \text{...(iii)}$$

∴ System of equations becomes

$$x + y + z = 12$$

$$2x + 3y + 3z = 33$$

and $$x - 2y + z = 0 \quad \textbf{(1)}$$

In matrix form, it can be written as

$$AX = B \quad \Rightarrow \quad X = A^{-1}B \quad \text{...(iv)}$$

where, matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

Now, $|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$$= 1(3 + 6) - 1(2 - 3) + 1(-4 - 3)$$

$$= 9 + 1 - 7 = 10 - 7 = 3 \neq 0 \quad \textbf{(1)}$$

So, A^{-1} exists.

Cofactors of $|A|$ are

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 3+6 = 9$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2-3 = 1$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4-3 = -7$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = 1+2 = -3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2-1 = 3$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 3-3 = 0$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$$

$$\therefore \quad C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix} \quad \textbf{(1)}$$

Now, $\text{adj}(A) = C^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|} = \dfrac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$ **(1/2)**

Now, from Eq. (iv), we have

$X = A^{-1}B$

$X = A^{-1}B$

$$X=\frac{1}{3}\begin{bmatrix}9 & -3 & 0\\ 1 & 0 & -1\\ -7 & 3 & 1\end{bmatrix}\begin{bmatrix}12\\ 33\\ 0\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}108-99\\ 12\\ -84+99\end{bmatrix}=\frac{1}{3}\begin{bmatrix}9\\ 12\\ 15\end{bmatrix}=\begin{bmatrix}3\\ 4\\ 5\end{bmatrix}$$

$$\Rightarrow \quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 4\\ 5\end{bmatrix}$$

On comparing corresponding elements, we get

$x=3,\ \ y=4$ and $z=5$ **(1½)**

The management of the colony must include the bravery award for some of its members. Because in this category, we appreciate the brave members of the colony for their bravery and make aware the other members (men, women and children) of the colony. **(1)**

9. Using matrices, solve the following system of equations.

$$x-y+2z=7$$
$$3x+4y-5z=-5$$
and $$2x-y+3z=12$$

Delhi 2012

Given, system of equations can be written in matrix form $AX = B$. So firstly, determine the cofactors of A and then determine A^{-1} and then use the relation $X = A^{-1}B$, to get the values of x, y and z.

Given, system of is equations are

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
and
$$2x - y + 3z = 12$$

In matrix form, it can be written as

$X = AB \Rightarrow X = A^{-1}B$ ...(i) **(1)**

where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Now, $|A| = 1(12-5) + 1(9+10) + 2(-3-8)$

$= 1(7) + 1(19) + 2(-11)$ **(1)**

$= 7 + 19 - 22 = 4$

$\Rightarrow$ $|A| \neq 0$, hence A^{-1} exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 1(12-5) = 7$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -1(9+10) = -19$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3-8) = -11$$

$$A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -1(-3+2) = 1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4) = -1$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1(-1+2) = -1$$

$$A_{31} = (-1)^4 \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 1(5-8) = -3$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -1(-5-6) = 11$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 1(4+3) = 7$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$ **(1½)**

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$ **(1/2)**

On putting the value of X, A^{-1} and B in Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

On comparing corresponding elements

On comparing corresponding elements, we get

$$x = 2,\ y = 1 \text{ and } z = 3 \quad \textbf{(1)}$$

10. Using matrices, solve the following system of linear equations.

$$x + y - z = 3$$
$$2x + 3y + z = 10$$
and $$3x - y - 7z = 1$$ **All India 2012**

Given system of equations is

$$x + y - z = 3;\ 2x + 3y + z = 10$$

and $3x - y - 7z = 1$

In matrix form, it can be written as

$$AX = B \Rightarrow X = A^{-1}B \quad \text{...(i)} \ \textbf{(1)}$$

where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Now, $|A| = 1(-21+1) - 1(-14-3) \cdot 1(-2-9)$
$= 1(-20) - 1(-17) - 1(-11)$ **(1)**
$= -20 + 17 + 11 = 8$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists.

Cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} = 1(-21+1) = -20$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} = -1(-14-3) = 17$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = 1(-2-9) = -11$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ -1 & -7 \end{vmatrix} = -1(-7-1) = 8$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 3 & -7 \end{vmatrix} = 1(-7+3) = -4$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \end{vmatrix} = -1(-1-3) = 4$$

$$\begin{vmatrix} 3 & -1 \end{vmatrix}$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1(1+3) = 4$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -1(1+2) = -3$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1(3-2) = 1 \qquad \textbf{(1½)}$$

$$\therefore \text{adj}(A) = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^T$$

$$= \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \textbf{(1)}$$

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$\Rightarrow \quad A^{-1} = \frac{1}{8} \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \qquad \textbf{(1/2)}$$

Now, from Eq. (i), we have

$$X = \frac{1}{8} \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -60+80+4 \\ 51-40-3 \\ -33+40+1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 3,\ y = 1 \text{ and } z = 1 \qquad \textbf{(1)}$$

11. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the system of equations

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

and $$x - 3y + z = 4.$$

Delhi 2012C

Given, $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$

$\because \quad |A| = 1(1+3) - 2(-1-1) + 1(3-1)$
$= 4 + 4 + 2 = 10$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix} = 1 + 3 = 4$$

$$A_{12} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = 2$$

$$A_{13} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} = -1(-3-2) = 5$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = -(1+1) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \quad \textbf{(1½)}$$

Now, $A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$$\therefore \quad A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \quad \textbf{(1/2)}$$

Given system of equations, can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \textbf{(1)}$$

$$\therefore X = A^{-1}B = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 16 + 0 + 4 \\ 8 + 0 + (-8) \\ 8 + 0 + 12 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

(1)

On comparing corresponding elements, we get

$x = 2, y = 0$ and $z = 2$ **(1)**

12. Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and then use to solve the system of equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$

and $2x + y + 3z = 1$. **Delhi 2012C; HOTS**

> Firstly, find the product of given matrices and then pre-multiply both sides of the product by A^{-1} and obtain A^{-1}. Then, by using A^{-1} and concept of matrix method, find the values of x, y and z.

Let $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$=\begin{bmatrix}-4+4+8 & 4-8+4 & -4-8+12\\ -7+1+6 & 7-2+3 & -7-2+9\\ 5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}$$

$$=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}=8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=8I \quad \textbf{(1½)}$$

$\Rightarrow$ $$BA=8I \Rightarrow B(AA^{-1})=8IA^{-1}$$

[multiplying both sides by A^{-1}]

$\Rightarrow$ $$B=8A^{-1} \qquad [\because AA^{-1}=I]$$

$\therefore$ $$A^{-1}=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix} \quad \textbf{(1)}$$

Given system of equations can be written in matrix form as

$$AX=C \Rightarrow X=A^{-1}C \quad \textbf{(1)}$$

where,

$$A=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}, C=\begin{bmatrix}4\\ 9\\ 1\end{bmatrix} \text{ and } X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

(1/2)

$\therefore$ $$X=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{8}\begin{bmatrix}-16+36+4\\ -28+9+3\\ 20-27-1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}24\\ -16\\ -8\end{bmatrix}=\begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$$

(1)

On comparing corresponding elements, we get

$$x=3, y=-2 \text{ and } z=-1 \quad \textbf{(1)}$$

13. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, solve the system of equations,

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
and $$3x - 3y - 4z = 11.$$

All India 2012C, 2010, 2008

Given, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$\therefore \quad |A| = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$
$= -6 + 28 + 45 = 67$ **(1)**

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1/2)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} = -12 + 6 = -6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} = -(1)(-8 - 6) = 14$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} = -6 - 9 = -15$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} = -(-8 - 9) = 17$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = (-4 + 9) = 5$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = -(-3 - 6) = 9$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (4 + 9) = 13$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & -3 \end{vmatrix} = -(2 + 6) = -8$$

$$A_{32} = (-1) \begin{vmatrix} & \\ 2 & 2 \end{vmatrix} = -(2+6) = -8$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (3-4) = -1$$

$$\therefore \ \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T \qquad \textbf{(1)}$$

$$= \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} \cdot \text{adj}(A)$

$$\therefore \qquad A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \qquad \textbf{(1/2)}$$

Given system of equations can be written in matrix form as

$$AX = B \quad \Rightarrow X = A^{-1}B$$

where,

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, \ B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \textbf{(1)}$$

$$\therefore \quad X = A^{-1}B = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ +60 + 18 - 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad \textbf{(1)}$$

On comparing corresponding elements, we get

$$x = 3, y = -2 \text{ and } z = 1 \quad \textbf{(1)}$$

14. Using matrix method, solve the following system of equations.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

and $$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, x, y, z \neq 0$$

Delhi 2011

Since, $|A| \neq 0$, so unique solution exists.

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} = 1(120 - 45) = 75$$

$$A_{12} = (-1)^3 \begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = -1(-80 - 30) = 110$$

$$A_{13} = (-1)^4 \begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} = 1(36 + 36) = 72$$

$$A_{21} = (-1)^3 \begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = -1(-60 - 90) = 150$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 10 \\ \end{vmatrix} = 1(-40 - 60) = -100$$

$$\begin{vmatrix} 6 & -20 \end{vmatrix}$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = -1(18-18) = 0$$

$$A_{31} = (-1)^4 \begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = 1(15+60) = 75$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = -1(10-40) = 30$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = 1(-12-12) = -24$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad \textbf{(1)}$$

Then, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \textbf{(1/2)}$$

On putting the values X, A^{-1} and B in Eq. (ii), we get

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

On comparing corresponding elements, we get

$$u = \frac{600}{1200}, \; v = \frac{400}{1200}, w = \frac{240}{1200} \quad \textbf{(1)}$$

$$\therefore \quad u = \frac{1}{2}, v = \frac{1}{3} \text{ and } w = \frac{1}{5}$$

But $\quad \frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3} \text{ and } \frac{1}{z} = \frac{1}{5}$$

$$\therefore \quad x = 2, y = 3 \text{ and } z = 5 \quad \textbf{(1)}$$

15. Using matrices, solve the following system of equations.

$$4x + 3y + 2z = 60$$

$$x + 2y + 3z = 45$$

and $\quad 6x + 2y + 3z = 70$ **All India 2011**

The given system of equations can be written in matrix form as

$$AX = B$$

where, $A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Its solution is given by

$$X = A^{-1}B \quad \text{...(i)}$$

where, $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$ **(1)**

Now, $|A| = 4 \times \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} - 3 \times \begin{vmatrix} 1 & 3 \\ 6 & 3 \end{vmatrix} + 2 \times \begin{vmatrix} 1 & 2 \\ 6 & 2 \end{vmatrix}$

$= 4(6 - 6) - 3(3 - 18) + 2(2 - 12)$ **(1)**

$= 4(0) - 3(-15) + 2(-10) = 0 + 45 - 20$

$= 25$

$\Rightarrow$ $|A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$A_{11} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 1(6 - 6) = 0$

$A_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 6 & 3 \end{vmatrix} = -1(3 - 18) = -1(-15) = 15$

$A_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 6 & 2 \end{vmatrix} = 1(2 - 12) = 1(-10) = -10$

$A_{21} = (-1)^3 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = -1(9 - 4) = -1(5) = -5$

$A_{22} = (-1)^4 \begin{vmatrix} 4 & 2 \\ 6 & 3 \end{vmatrix} = 1(12 - 12) = 0$

$A_{23} = (-1)^5 \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = -1(8 - 18) = -1(-10) = 10$

$A_{31} = (-1)^4 \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 1(9 - 4) = 1(5) = 5$

$$A_{32} = (-1)^5 \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = -1(12-2) = -1(10) = -10$$

$$A_{33} = (-1)^6 \begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix} = 1(8-3) = 5$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 15 & -10 \\ -5 & 0 & 10 \\ 5 & -10 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$ **(1½)**

$$\text{Then, } A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

(1/2)

From Eq. (i), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$= \frac{1}{25}\begin{bmatrix} 0 - 225 + 350 \\ 900 + 0 - 700 \\ -600 + 450 + 350 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 5, y = 8$ and $z = 8$ **(1)**

16. Using matrices, solve the following system of equations.

$$x + 2y + z = 7$$
$$x + 3z = 11$$
and $$2x - 3y = 1$$

All India 2011; Delhi 2008C

The given system of equations can be written in matrix form as $AX = B$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}, B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Its solution is given by

$$X = A^{-1}B \quad \text{...(i)} \quad \textbf{(1)}$$

Now,

$$|A| = 1 \times \begin{vmatrix} 0 & 3 \\ -3 & 0 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 0 \\ 2 & -3 \end{vmatrix}$$

$$= 1(0 + 9) - 2(0 - 6) + 1(-3 - 0)$$

$$= 9 + 12 - 3 = 18$$

$\Rightarrow \quad |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 0 & 3 \\ -3 & 0 \end{vmatrix} = 1(0 + 9) = 9$$

$$A_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = -1(0 - 6) = -1(-6) = 6$$

$$A_{13} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 2 & -3 \end{vmatrix} = 1(-3 - 0) = -3$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix} = -1(0 + 3) = -1(3) = -3$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 1(0 - 2) = 1(-2) = -2$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$= -1(-3-4) = -1(-7) = 7$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 1(6-0) = 1(6) = 6$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -1(3-1) = -1(2) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 1(0-2) = 1(-2) = -2$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$ **(1½)**

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

(1/2)

From Eq. (i), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

On comparing corresponding elements,
$x = 2, y = 1$ and $z = 3$. **(1)**

17. Using matrices, solve the following system of equations.

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
and $$3x - 3y - 4z = 11.$$ **All India 2011, 2008**

17. Using matrices, solve the following system of equations.

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
and $$3x - 3y - 4z = 11.$$ **All India 2011, 2008**

The given system of equations can be written in matrix form as $AX = B$,

where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Its solution is given by, $X = A^{-1}B$ ...(i) **(1)**

Now, $|A| = 1(-12+6) - 2(-8-6) - 3(-6-9)$

$= 1(-6) - 2(-14) - 3(-15)$

$= -6 + 28 + 45 = 67$

$\Rightarrow$ $|A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 3 & 2 \\ -3 & -4 \end{vmatrix} = 1(-12+6) = -6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 2 & 2 \\ 3 & -4 \end{vmatrix} = -1(-8-6)$$

$$= -1(-14) = 14$$

$$A_{13} = (-1)^4 \begin{vmatrix} 2 & 3 \\ 3 & -3 \end{vmatrix} = 1(-6-9)$$

$$= 1(-15) = -15$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & -3 \\ -3 & -4 \end{vmatrix} = -1(-8-9)$$

$$= -1(-17) = 17$$

$$A_{22} = (-1)^4 \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = 1(-4+9) = 1(5) = 5$$

$$A_{23} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -3 \end{vmatrix} = -1(-3-6)$$

$$= -1(-9) = 9$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 1(4+9) = 13$$

$$A_{32} = (-1)^5 \begin{vmatrix} 1 & -3 \\ 2 & 2 \end{vmatrix} = -1(2+6)$$

$$= -1(8) = -8$$

$$A_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1(3-4)$$

$$= 1(-1) = -1$$

$$\therefore \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ **(1½)**

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

(1/2)

From Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$ **(1)**

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix}$$

$$= \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$\Rightarrow x = 3, y = -2$ and $z = 1$ **(1)**

$\Rightarrow \quad x = 3, y = -2$ and $z = 1$ **(1)**

18. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

to solve the system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$

and $\quad 3x - 2y + 4z = 2$ **HOTS; Foreign 2011**

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now, $AC = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(1½)**

$\Rightarrow \quad AC = I$

Now, on pre-multiplying both sides by A^{-1}, we get

$$A^{-1}AC = A^{-1}I$$

$\Rightarrow \quad IC = A^{-1}$

$[\because A^{-1}A = I$ and $A^{-1}I = A^{-1}]$

$\Rightarrow \quad C = A^{-1}$ **(1)**

$\therefore A^{-1} = C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ **(1/2)**

Now, given system of equations can be written as

$$AX = B$$

where, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

The solution of system of equation is given by

$$X = A^{-1}B$$

$\Rightarrow \quad X = CB \quad [\because A^{-1} = C]$ **(1)**

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ **(1)**

$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$

On comparing corresponding elements, we get

$x = 0, y = 5$ and $z = 3$ **(1)**

19. If $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations

$$2x - y + z = -3$$
$$3x - z = 0$$
and $\quad 2x + 6y - z = 2$ **All India 2011C**

Given, $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$

Now, $|A| = 2(0+6) + 1(0+2) + 1(18-0)$

$$= 2(6) + 1(2) + 1(18)$$
$$= 12 + 2 + 18 = 32$$

$\Rightarrow$ $|A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 \begin{vmatrix} 0 & -1 \\ 6 & 0 \end{vmatrix} = 1(0+6) = 6$$

$$A_{12} = (-1)^3 \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} = -1(0+2)$$
$$= -1(2) = -2$$

$$A_{13} = (-1)^4 \begin{vmatrix} 3 & 0 \\ 2 & 6 \end{vmatrix} = 1(18-0)$$
$$= 1(18) = 18$$

$$A_{21} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 6 & 0 \end{vmatrix} = -1(0-6)$$
$$= -1(-6) = 6$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = 1(0-2) = -2$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & -1 \\ 2 & 6 \end{vmatrix} = -1(12+2) = -14$$

$$A_{31} = (-1)^4 \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = 1(1-0) = 1$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -1(-2-3) = 5$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = 1(0+3) = 3$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -2 & 18 \\ 6 & -2 & -14 \\ 1 & 5 & 3 \end{bmatrix}' \qquad \textbf{(1½)}$$

$$= \begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix} \qquad \textbf{(1)}$$

$$\therefore \quad A^{-1} = \frac{1}{32}\begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix} \qquad \textbf{(1/2)}$$

Now, given system of equation can be written as $Ax = B$

where, $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $\qquad B = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$

The solution of system of equation is given by

$$X = A^{-1}B$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{32}\begin{bmatrix} 6 & 6 & 1 \\ -2 & -2 & 5 \\ 18 & -14 & 3 \end{bmatrix}\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} \qquad \textbf{(1)}$$

$$= \frac{1}{32}\begin{bmatrix} -18+0+2 \\ 6-0+10 \\ -54-0+6 \end{bmatrix} = \frac{1}{32}\begin{bmatrix} -16 \\ 16 \\ -48 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ -3/2 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = -\frac{1}{2}, y = \frac{1}{2} \text{ and } z = -\frac{3}{2} \qquad \textbf{(1)}$$

20. If $A=\begin{bmatrix}1 & -2 & 1\\ 0 & -1 & 1\\ 2 & 0 & -3\end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$x-2y+z=0$$
$$-y+z=-2$$
and $$2x-3z=10$$ **All India 2011C**

Given, $A=\begin{bmatrix}1 & -2 & 1\\ 0 & -1 & 1\\ 2 & 0 & -3\end{bmatrix}$

Now, $|A|=1(3-0)+2(0-2)+1(0+2)$
$=1(3)+2(-2)+1(2)=3-4+2=1$

$\Rightarrow |A|\neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11}=(-1)^2(3-0)=1\times 3=3$$
$$A_{12}=(-1)^3(0-2)=-1\times -2=2$$
$$A_{13}=(-1)^4(0+2)=1\times 2=2$$
$$A_{21}=(-1)^3(6-0)=-1\times 6=-6$$
$$A_{22}=(-1)^4(-3-2)=1\times -5=-5$$
$$A_{23}=(-1)^5(0+4)=-1\times 4=-4$$
$$A_{31}=(-1)^4(-2+1)=1\times -1=-1$$
$$A_{32}=(-1)^5(1-0)=-1\times 1=-1$$
$$A_{33}=(-1)^6(-1+0)=1\times -1=-1$$

$$\therefore \text{adj}(A)=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}3 & 2 & 2\\ -6 & -5 & -4\\ \end{bmatrix}^T=\begin{bmatrix}3 & -6 & -1\\ 2 & -5 & -1\end{bmatrix}$$ **(2½)**

$$\begin{bmatrix} -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} 2 & -4 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix} \qquad \textbf{(1/2)}$$

[here, $|A| = 1$]

Now, given system of equations can be witten as

$$AX = B$$

where, $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $\quad B = \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$

whose solution is given by

$$X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ 2 & -4 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 12 - 10 \\ 0 + 10 - 10 \\ 0 + 8 - 10 \end{bmatrix} \qquad \textbf{(1)}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

On comparing corresponding elements, we get

$$x = 2, y = 0 \text{ and } z = -2 \qquad \textbf{(1)}$$

21. If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$,

then find AB and hence solve system of equations

$$x-2y=10$$
$$2x+y+3z=8$$
and $$-2y+z=7$$ **Delhi 2011C**

Given,

$$A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Firstly, we find product AB and then use it to find inverse A^{-1}. **(1)**

Now, $$AB=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

$$=\begin{bmatrix}7+4-0 & 2-2+0 & -6+6+0\\ 14-2-12 & 4+1+6 & -12-3+15\\ 0+4-4 & 0-2+2 & 0+6+5\end{bmatrix}$$

$$\Rightarrow \quad AB=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}=11I$$ **(1½)**

$$\Rightarrow \quad AB=11I$$

On pre-multiplying both sides of Eq. (i) by A^{-1}, we get

$$A^{-1}AB=11A^{-1}I \Rightarrow IB=11A^{-1}$$
$$[\because A^{-1}A=I \text{ and } A^{-1}I=A^{-1}]$$

$$\Rightarrow \quad B=11A^{-1} \quad [\because IB=B]$$

$$\Rightarrow \quad A^{-1}=\frac{1}{11}\cdot B$$

$$\Rightarrow \quad A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \qquad \textbf{(1½)}$$

Now, given system of equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

whose solution is given by $X = A^{-1}B$.

$$\Rightarrow X = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \qquad \textbf{(1)}$$

$$= \frac{1}{11}\begin{bmatrix} 70+16-42 \\ -20+8-21 \\ -40+16+35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 4, y = -3$ and $z = 1$. **(1)**

22. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$3x - 4y + 2z = -1$$
$$2x + 3y + 5z = 7$$

and $\qquad x + z = 2$ **Delhi 2011C**

Given, $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

Now, $|A| = 3(3-0) + 4(2-5) + 2(0-3)$

$= (3 \times 3) + (-4 \times 3) + (2 \times -3)$

$= 9 - 12 - 6 = 9 - 18 = -9$

$\Rightarrow |A| \neq 0$, hence unique solution exists. (1)

$\Rightarrow |A| \neq 0$, hence unique solution exists. **(1)**

Now, cofactors of $|A|$ are

$$A_{11} = (-1)^2 (3-0) = 1 \times 3 = 3$$
$$A_{12} = (-1)^3 (2-5) = -1 \times -3 = 3$$
$$A_{13} = (-1)^4 (0-3) = 1 \times -3 = -3$$
$$A_{21} = (-1)^3 (-4-0) = -1 \times -4 = 4$$
$$A_{22} = (-1)^4 (3-2) = 1 \times 1 = 1$$
$$A_{23} = (-1)^5 (0+4) = -1 \times 4 = -4$$
$$A_{31} = (-1)^4 (-20-6) = 1 \times -26 = -26$$
$$A_{32} = (-1)^5 (15-4) = -1 \times 11 = -11$$
$$A_{33} = (-1)^6 (9+8) = 1 \times 17 = 17$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -11 & 17 \end{bmatrix}^T \quad \textbf{(1½)}$$

$$= \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \quad \textbf{(1)}$$

$$A^{-1} = -\frac{1}{9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \quad \textbf{(1/2)}$$

Now, given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

'hose solution is given by $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix} \quad \textbf{(1)}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -3+28-52 \\ -3+7-22 \\ 3-28+34 \end{bmatrix} = -\frac{1}{9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

On comparing corresponding elements, we get

$x = 3, y = 2$ and $z = -1.$ **(1)**

23. If $A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations

$$8x - 4y + z = 5$$
$$10x + 6z = 4$$

and $$8x + y + 6z = \frac{5}{2}$$ **All India 2010C**

Do same as Que. 22.

$$\left[\textbf{Ans. } A^{-1} = \frac{1}{10}\begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}\right.$$

$$\left.\text{and } x = 1, y = \frac{1}{2}, z = -1\right]$$

24. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$,

then find AB. Use this to solve the system of equations

$$x-y=3$$
$$2x+3y+4z=17$$
and $$y+2z=7$$ **All India 2010C**

Do same as Que. 21.

[**Ans.** $AB=6I$ and $x=2, y=-1, z=4$]

25. If $A=\begin{bmatrix}3 & 2 & 1\\ 4 & -1 & 2\\ 7 & 3 & -3\end{bmatrix}$, then find A^{-1}. Hence,

solve the following system of equations

$$3x+2y+z=6$$
$$4x-y+2z=5$$
and $$7x+3y-3z=7$$ **Delhi 2010C**

Do same as Que. 22.

[**Ans.** $A^{-1}=\frac{1}{62}\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 5 & 5 & -11\end{bmatrix}$

and $x=1, y=1, z=1$]

26. Using matrices, solve system of linear equations

$$x+y+z=6$$
$$x+2z=7$$
and $$3x+y+z=12$$ **All India 2009**

Do same as Que. 17.

[**Ans.** $x=3, y=1$ and $z=2$]

27. Solve the following system of equations, by using matrix method.

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
and $$4x - 3y + 2z = 4$$ **Foreign 2009**

Do same as Que. 17.

[**Ans.** $x = 1, y = 2$ and $z = 3$]

28. Using matrices, solve the following system of equations

$$x + y + z = 1$$
$$x - 2y + 3z = 2$$
and $$x - 3y + 5z = 3$$ **All India 2009C**

Do same as Que. 17.

$\left[\textbf{Ans. } x = \frac{1}{2}, y = 0 \text{ and } z = \frac{1}{2}\right]$.

29. Using matrices, solve the following system of equations

$$8x + 4y + 3z = 18$$
$$2x + y + z = 5$$
and $$x + 2y + z = 5$$

All India 2009C

Do same as Que. 17.

[**Ans.** $x = 1, y = 1$ and $z = 2$]

30. Using matrices, solve the following system of equations

$$x + y - z = 3$$
$$2x + 3y + z = 10$$
and $$3x - y - 7z = 1$$ **Delhi 2009C**

Do same as Que. 17.

[**Ans.** $x = 3, y = 1$ and $z = 1$]

31. Using matrices, solve the following system of equations

$$2x + 8y + 5z = 5$$
$$x + y + z = -2$$
and $$x + 2y - z = 2$$ **Delhi 2009C**

Do same as Que. 17.

[**Ans.** $x = -3, y = 2$ and $z = -1$]

32. Using matrices, solve the following system of equations

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
and $$3x - 2y + 4z = 2$$ **All India 2008C**

Do same as Que. 17.

[**Ans.** $x = -3, y = 2$ and $z = -1$]

33. Using matrices, solve the following system of equations

$$2x + y + z = 7$$
$$x - y - z = -4$$
and $$3x + 2y + z = 10$$ **All India 2008C**

Do same as Que. 17.

[**Ans.** $x = 1, y = 2$ and $z = 3$]